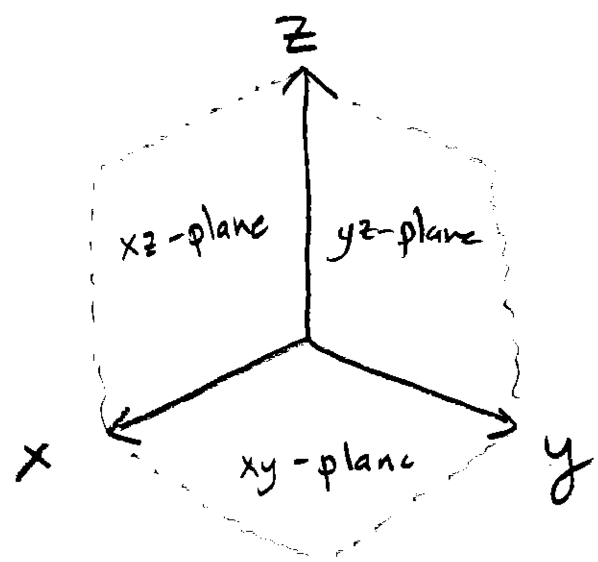


126: Calculus III - Dr. Andy Loveless

12.1 Intro to 3D

Entry Task:

A) How can you tell if a point (x,y,z) in $\mathbb{R}^3$ is on…

1. …the xy-plane? $\iff$ $z = 0$ $(x, y, 0)$
2. …the yz-plane? $\iff$ $x = 0$ $(0, y, z)$
3. …the xz-plane? $\iff$ $y = 0$ $(x, 0, z)$
4. …the z-axis? $\iff$ $x = 0$ AND $y = 0$ $(0, 0, z)$
5. …the y-axis? $\iff$ $x = 0$ AND $z = 0$ $(0, y, 0)$
6. …the x-axis? $\iff$ $y = 0$ AND $z = 0$ $(x, 0, 0)$
7. …the origin? $\iff$ $x = 0, y = 0, z = 0$ $(0, 0, 0)$

Observations

Basic Planes

SET NOTATION

ASIDE

$z = 3 \iff$ PLANE PARALLEL TO xy-PLANE BUT 3 UNITS UP.

xy-plane $\Leftrightarrow \{(x, y, z) \mid \; z = 0\} \Leftrightarrow z = 0$

yz-plane $\Leftrightarrow \{(x, y, z) \mid \; x = 0\} \Leftrightarrow x = 0$

"SUCH THAT"

xz-plane $\Leftrightarrow \{(x, y, z) \mid \; y = 0\} \Leftrightarrow y = 0$

READ: "ALL POINTS (x, y, z) SUCH THAT $y = 0$"

Basic Lines

ASIDE

$\underbrace{x = 1,\ y = 3,\ z = \text{anything}}$

LINE PARALLEL TO z-AXIS AND THRU $(1, 3, 0)$

x-axis $\Leftrightarrow \{(x, y, z) \mid y = 0 \textbf{ and } z = 0\}$

y-axis $\Leftrightarrow \{(x, y, z) \mid x = 0 \textbf{ and } z = 0\}$

z-axis $\Leftrightarrow \{(x, y, z) \mid x = 0 \textbf{ and } y = 0\}$

NOTE

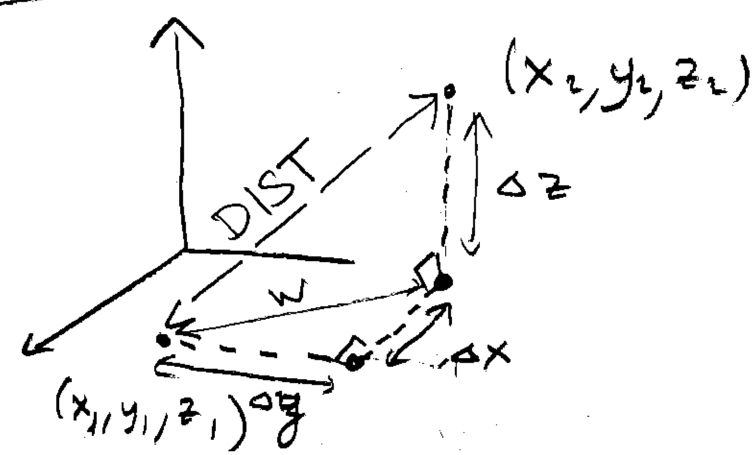
$x = 1, y = 3$ IS A POINT IN $\mathbb{R}^2$

$x = 1, y = 3$ IS A LINE IN $\mathbb{R}^3$

ASIDE DERIVATION

Distances: The distance (in a straight ine) between two points in $\mathbb{R}^3$ is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$



$w^2 = (\Delta x)^2 + (\Delta y)^2$

AND $w^2 + (\Delta z)^2 = \text{DIST}^2$

$\Rightarrow \text{DIST} = \sqrt{w^2 + (\Delta z)^2}$

$= \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$

How far is (1,3,4) from…

1. …the origin?
2. …the xy-plane?
3. …the x-axis?

1] (1,3,4) TO (0,0,0)

$\sqrt{(1-0)^2+(3-0)^2+(4-0)^2} = \sqrt{1+9+16} = \sqrt{26}$

2] (1,3,4) TO (1,3,0)

$\sqrt{(1-1)^2+(3-3)^2+(4-0)^2} = 4$ SHOULD MAKE SENSE, DIDN'T NEED FORMULA!

3] (1,3,4) TO (1,0,0)

$\sqrt{(1-1)^2+(3-0)^2+(4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Homework Hints

There is a way to answer the following questions using only the distance formula:

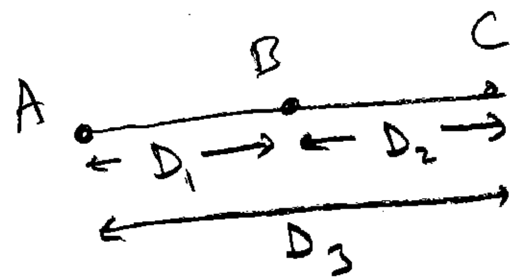
Given three points

$A(a_1, a_2, a_3), B(b_1, b_2, b_3), C(c_1, c_2, c_3)$

1. Are the points on the same line?

2. Do the points form a right triangle?

$\boxed{1}$



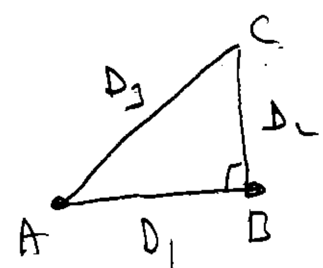
FIND $|AB| = D_1$

$|BC| = D_2$

$|AC| = D_3$

IF BIGGEST = "SUM OF OTHER TWO"
THEN YES ON SAME LINE

$\boxed{2}$



IF
$D_1^2 + D_2^2 = D_3^2$
THEN RIGHT TRIANGLE YES

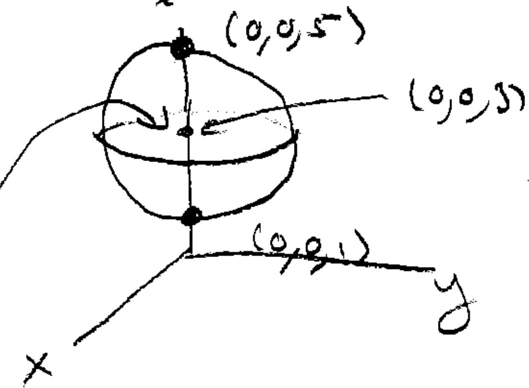
IF $D_1^2 + D_2^2 \neq D_3^2$ THEN NO

Spheres *(HW 12.1/6-16)*

The equation of all points (x, y, z) on a sphere (*i.e.* the outer shell of a ball) centered at (h, k, l) with radius r is

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

← DISTANCE FORMULA



Example: Find the equation of the sphere that has its lowest point at (0,0,1) and its highest point at (0,0,5).

CENTER = (0, 0, 3)

RADIUS = 2

$$\boxed{x^2 + y^2 + (z-3)^2 = 2^2}$$

Example:

Describe the intersection of the sphere $x^2 + y^2 + (z-3)^2 = 4$ and the $\underbrace{xz}_{y=0}$-plane.

$$\left.\begin{aligned} x^2+y^2+(z-3)^2 &= 4 \\ y &= 0 \end{aligned}\right\} \text{ INTERSECTION ?}$$

$$x^2+0^2+(z-3)^2=4$$

$$x^2+(z-3)^2=4 \leftarrow \text{CIRCLE!!!}$$

$$\boxed{\{(x,y,z) \mid y=0 \text{ AND } x^2+(z-3)^2=4\}}$$

CIRCLE ON xz-plane

CENTERED AT $x=0, z=3$ OF RADIUS 2

What if it was the $\underbrace{xy}_{z=0}$-plane?

$$\left.\begin{aligned} x^2+y^2+(z-3)^2 &= 4 \\ z &= 0 \end{aligned}\right\} \text{ INTERSECTION ?}$$

$$x^2+y^2+(0-3)^2=4$$

$$x^2+y^2+9=4$$

$$x^2+y^2=-5 \leftarrow \text{NO POINTS!!}$$

NO INTERSECTION

("DNE" IN HW)

Example: Find the center and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 = 26 + 12x$$

$\div 2$

$$x^2 + y^2 + z^2 = 13 + 6x$$

$-6x$

$$x^2 - 6x \quad + y^2 + z^2 = 13$$

COMPLETE SQUARE $\rightarrow$ HALF MIDDLE $= -3 \rightarrow$ SQUARE $= 9$

$$\underbrace{x^2 - 6x + 9} - 9 + y^2 + z^2 = 13$$

$$(x-3)^2 - 9 + y^2 + z^2 = 13$$

$$(x-3)^2 + y^2 + z^2 = 22$$

$$\boxed{\begin{array}{l} \text{CENTER} = (3,0,0) \\ \text{RADIUS} = \sqrt{22} \end{array}}$$